are:

$$u_{p_B} = \Omega\cos\theta[R_s\cos(\psi-\eta) - y_B\cos\phi + z_B\sin(\psi-\eta)] - w_{c.g.}\sin\theta$$

$$v_{p_B} = \Omega R_s[\sin\phi\sin\theta\cos(\psi-\eta) - \cos\phi\sin(\psi-\eta)] + \Omega x_B\cos\phi\cos\theta + \Omega z_B\sin\theta + w_{c.g.}\sin\phi\cos\theta$$

$$w_{p_B} = \Omega R_s[\cos\phi\sin\theta\cos(\psi-\eta) + \sin\phi\sin(\psi-\eta)] - \Omega x_B\sin\phi\cos\theta - \Omega y_B\sin\theta + w_{c.g.}\cos\phi\cos\theta \quad (1)$$

and the corresponding polar-coordinate representations are,

$$\begin{bmatrix} V \\ \alpha \\ \beta \end{bmatrix}_p = \begin{bmatrix} (u_{p_B}^2 + v_{p_B}^2 + w_{p_B}^2)^{1/2} \\ \tan^{-1}(w_{p_B}/u_{p_B}) \\ \sin^{-1}(v_{p_B}/V) \end{bmatrix} \quad (2)$$

Aircraft Orientation

Data for five spin modes resulting from various tail modifications are given in Ref. 3. Flight variables referenced to the c.g. are presented in Table 1. The variables θ and ϕ are found from:

$$\theta = \sin^{-1}(-p/\Omega) \qquad \phi = \tan^{-1}(q/r) \quad (3)$$

R_s is found by: $R_s = \sqrt{V^2 - w_{c.g.}^2}\,(1/\Omega)$. V and $\alpha_{c.g.}$ are given, and $\psi - \eta$ is found numerically. These values also are presented in Table 1.

Referring to Table 1, it is found that $\psi - \eta$ is greater than 90 deg for all spin modes: i.e., the aircraft's nose not only points toward the center of the spin, but lags behind it even though $\beta_{c.g.}$ is considerably smaller than $\psi - \eta$. The velocity in the horizontal direction v_H (Fig. 1) is much less than the vertical velocity $w_{c.g.}$; therefore, the primary motion is downward, not circular, and the results are consistent. (Similar results are presented for high-performance aircraft in Ref. 4.) The spin radii R_s is within the wing span, as might be anticipated.[5]

Geometric Description of the Flow

Given the data of Table 1 and the equations derived above, it is possible to plot contours of constant V, α, and β over the aircraft, such as those shown in Figs. 2 and 3. It should be realized when viewing these figures that the airplane is pitched, rolled, and yawed with respect to the inertial frame; consequently, the distributions of V, α, and β are somewhat complex.

The location of the center of the spin relative to the aircraft is inferred from the velocity magnitude distribution (Fig. 2a): the nonzero roll angle causes the deviation of the velocity curves from perfect circles. The pitch angle has a major effect on distributions in the aircraft's vertical plane (Fig. 3). The direction of the spin (Fig. 2a) is clockwise about the spin axis. The velocity ranges from about 28 m/s at the nose in spin 4b to about 42 m/s at the tail in spin 4a.

Lines of constant α are almost parallel to the fuselage and range in value from about 35 deg at the left wingtip in spin 4b to greater than 90 deg at the right wingtip in spin 4a. The very large variation of α over the wing span causes a large variation in lift and, therefore, in induced drag: the drag on the right (retreating) wing is much greater than on the left (advancing) wing. There is little variation of α in the vertical plane (Fig. 2b).

Lines of constant β are nearly perpendicular to the fuselage (Fig. 2c), and range from about -35 deg at the tail in spin 4b to about -5 deg at the nose in all the spins. The flow over the vertical tail tends to oppose the spin; however, at the very high angles of attack involved, the tail's restoring moment could be reduced by interference in the flow from the wings and fuselage. There is a large β variation in the vertical plane (Fig. 3c).

Driving Mechanism

A rudimentary application of strip theory[1] suggests that the driving mechanism for the spin is the differential induced drag on the aircraft's wings. For the analysis, it was assumed that the flow was two-dimensional and that the only significant contributions to the forces and moments came from the wings, horizontal tail, and vertical tail.

As mentioned above, the retreating wing is at a higher angle of attack and therefore has a higher induced drag than the advancing wing. This is what drives the spin. Opposing the spin is the side force (acting through a moment arm about the center of gravity) on the vertical tail. This result is consistent with the qualitative guidance provided by Ref. 5.

Conclusions

This Note has presented a preliminary analysis of the geometric properties of a fully evolved spin and their effects on the flowfield about a general aviation aircraft. The results could form the basis for an analytical study of the forces and moments acting on a spinning aircraft, and they provide insights regarding the underlying causes of the spin.

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Errata

Reply by Author to P. R. Payne

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THE references cited in the first paragraph are incorrect. Reference 6 should be Ref. 7 and Ref. 7 should be Ref. 8.

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